

TQFT AND CATEGORIFICATION

Schedule

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 10:30	Opening Lauda	Benard Wedrich	De Renzi Virelizier	Palmer-Anghel Levine	Gainutdinov (9:30)
10:30 11:00	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11:00 12:00	Abouzaid	Juhasz	Petkova	Ghiggini	Willis (11:45)
12:00 – 14:30	Lunch break	Lunch break	Lunch break	Lunch break	
14:30 15:30	Discussions	Discussions	Discussions	Discussions	
15:30 16:00	Coffee break	Coffee break	Coffee break	Coffee break	
16:00 17:00	Beliakova	Meusburger	Geer	Costantino	
17:00 18:00	Dowlin (17:45)	Putyra	Wagner	Masbaum (17:45)	
Special	Welcome			Dinner	

Abstracts

Mohammed Abouzaid: Symplectic knot invariants

I will give a survey of the construction of knot invariants using Floer theoretic methods, focusing on the case of the symplectic analogues of Khovanov homology.

Anna Beliakova: Quantum 3-manifold invariants and non-semisimple TQFTs

In the late 80th Reshetikhin and Turaev defined quantum 3-manifold invariants associated with a semi-simple Lie algebra and a positive integer r . These invariants generalise the colored Jones polynomial, but were not categorified yet. Hennings, Lyubashenko and Kerler further extended them to weak non-semisimple TQFTs. Recently, Hennings invariants were combined with the modified trace techniques due to Geer and Patureau to produce a stronger invariant. Already at small roots of unity, this invariant recovers the Reidemeister torsion, and hence, admit a categorification by the Heegaard Floer Homology package.

In the talk I will give a gentle introduction to quantum 3-manifold invariants.

Léo Bénard: Acyclic Reidemeister torsion and incompressible surfaces in hyperbolic knot complements

We are interested in the character variety $X(M)$ of a hyperbolic knot complement in a homology sphere M . On a component X that carries the character of a lift of the holonomy of the hyperbolic structure, there is a twisted Alexander polynomial studied by Dunfield, Friedl and Jackson that conjecturally contains many topological invariants of the knot. The acyclic Reidemeister torsion can be seen as a specialization of this polynomial. We study this function as the torsion of a twisted cohomological complex of M , and with the help of incompressible surfaces constructed via the Culler-Shalen theory, we give sufficient conditions for this function to have a pole at infinity. In particular it implies that the torsion, hence this polynomial, is non-constant.

François Costantino: Stated Skein algebras as extended TQFTs

In this talk I will report on a joint work in progress with Thang Le in which we study a new version of skein algebras of surfaces, called “stated skein algebras” which has been defined by Thang Le building upon previous works of Bonahon-Wong and Muller.

After recalling what skein algebras and stated skein algebras are, and recalling some of their main properties, I will explain why these objects are richer than the original algebras by showing that the quantum group $O_q(\mathrm{SL}_2)$ is a special case of these algebras and then re-interpreting Thang Le’s “splitting theorem” in terms of cotensor products of co-modules. In the last part of the talk I will speculate on how these rich structures fit into a general framework of an extended TQFT with suitable target category.

Marco De Renzi: Renormalized Hennings invariants and TQFT

Non-semisimple constructions in quantum topology produce strong invariants and TQFTs with unprecedented properties. The first family of non-semisimple quantum invariants of 3-manifolds was defined by Hennings in 1996. The construction enabled Lyubashenko to build mapping class groups representations out of every finite-dimensional factorizable ribbon Hopf algebra. Further attempts at extending these constructions to TQFTs only produced partial results, as the vanishing of Hennings invariants in many crucial situations made it impossible to treat non-connected surfaces. We will show how to overcome these problems. In order to do so, we will first renormalize Hennings invariants through the use of modified traces. When working with factorizable Hopf algebras, we further show that the universal construction of Blanchet, Habegger, Masbaum and Vogel produces a fully monoidal TQFT which extends Lyubashenko’s mapping class group representations. This is a joint work with Nathan Geer and Bertrand Patureau.

Nathan Dowlin: A spectral sequence on Khovanov homology

We define a family of invariants $HFK_n(L)$ for $n \geq 1$ in the knot Floer realm which share many properties with the \mathfrak{sl}_n homology of Khovanov and Rozansky. Based on computational evidence and structural properties, we conjecture that for all $n \geq 1$ and all links L in S^3 , there is a spectral sequence from the \mathfrak{sl}_n homology of L to $HFK_n(L)$. In the $n = 2$ case, reduced $HFK_2(K)$ is isomorphic to δ -graded knot Floer homology, so this corresponds to Rasmussen's conjectured spectral sequence from Khovanov homology to δ -graded knot Floer homology.

We also construct a purely algebraic complex $CFK_2^{alg}(L)$ inspired by $CFK_2(L)$. This complex comes equipped with a filtration, and the E_2 page is isomorphic to Khovanov homology, so this gives a spectral sequence from the Khovanov homology of K to $HFK_2^{alg}(L)$. Thus, proving the conjectured spectral sequence from Khovanov homology to δ -graded knot Floer homology would follow from an isomorphism between $CFK_2^{alg}(L)$ and $CFK_2(L)$. Unfortunately, isomorphisms of this type are notoriously difficult, as one needs to show that all holomorphic discs have been counted.

Azat Gainutdinov: Modified trace is a symmetrised integral

A modified trace for a finite k -linear pivotal category is a family of linear forms on endomorphism spaces of projective objects which has cyclicity and so-called partial trace properties. We show that a non-degenerate modified trace defines a compatible with duality Calabi-Yau structure on the subcategory of projective objects. The modified trace provides a meaningful generalisation of the categorical trace to non-semisimple categories and allows to construct interesting topological invariants. Our main theorem says that for any finite-dimensional unimodular pivotal Hopf algebra H , a modified trace is determined by a symmetric linear form on H constructed from an integral. More precisely, we prove that shifting with the pivotal element defines an isomorphism between the space of right integrals, which is known to be 1-dimensional, and the space of modified traces. This result allowed us to compute modified traces for all simply laced restricted quantum groups at roots of unity. This is a joint work with Anna Beliakova and Christian Blanchet.

Nathan Geer: Invariants of 3-manifolds from Lie super algebras

In this talk I will discuss how to construct topological invariants from the special linear Lie super algebra $\mathfrak{sl}(m, n)$. The usual Reshetikhin-Turaev 3-manifold invariant arises from a modular category. The standard way to produce a modular category is to mod out by the modules with zero quantum dimension at a root of unity. The category of finite dimensional quantum $\mathfrak{sl}(m, n)$ modules is not semi-simple and its quantum dimension vanishes generically (for any value of the quantum parameter). Thus, in this context the standard construction does not produce an interesting category. Instead, we define a non-zero modified dimension which vanishes for some modules at a root of unity. We show that modulo these modules the

category of finite dimensional quantum $\mathfrak{sl}(m, n)$ modules produces a non-trivial 3-manifold invariant. This is joint work with Cristina Palmer-Anghel and Bertrand Patureau.

Paolo Ghiggini: A natural set of generators for the wrapped Fukaya category of Weinstein domains

A Weinstein manifold is an open symplectic manifold admitting a proper Morse function adapted to the symplectic structure. It turns out that the critical points of such a function have Morse index at most half of the dimension of W . When the index is half the dimension, their unstable manifolds are Lagrangian discs. In a joint work with Baptiste Chantraine, Georgios Dimitroglou Rizell and Roman Golovko, we decompose any object in the wrapped Fukaya category of a Weinstein manifold as an iterated cone on the unstable manifolds of the half-dimensional critical points.

Andras Juhasz: Duality in the sutured Floer TQFT and Akbulut concordance surgery

We give an explicit construction of the Honda–Kazez–Matić gluing maps in terms of contact handles. We use this to prove a duality result for turning a sutured manifold cobordism around, and to compute the maps induced by the trace and cotrace cobordisms in the sutured Floer TQFT. We also show that the decorated link cobordism maps on the hat version of link Floer homology defined via sutured manifold cobordisms and via elementary cobordisms agree.

As an application, we compute the effect of a generalization of the Fintushel–Stern knot surgery operation using a self-concordance of a knot, due to Akbulut, on the Ozsváth–Szabó 4-manifold invariant. The formula involves the graded Lefschetz number of the concordance map on knot Floer homology, and the proof uses a version of sutured Floer homology perturbed by a 2-form. This is joint work with Ian Zemke.

Aaron Lauda: An introduction to quantum categorification

The "quantum" in quantum topology usually refers to invariants in low-dimensional topology that have a representation theoretic interpretation in the language of quantum groups. These invariants are also closely connected with constructions in theoretical physics emanating from Chern-Simons gauge theory and its generalizations. Representation theory provides the bridge whereby ideas motivated from theoretical physics can be mathematically formulated to study of low-dimensional topology. Even the elementary and combinatorial constructions of link invariants like the Jones polynomial via the Kauffman bracket can be understood as arising through a careful analysis of the representation theory of quantum groups.

Higher representation theory studies the next layer in representation theory by categorifying the objects that generate the symmetries of interest. Categorifying quantum groups and other symmetry algebras gives rise to richer invariants in low-dimensional topology. In this talk we will provide a primer on higher representation theory and survey some of the ways

it has been used in the study of link homology theory. We will also highlight some of the frontiers in the field and the most active areas of current research.

Adam Levine: Khovanov homology and knot Floer homology for pointed links

There are spectral sequences relating reduced Khovanov homology to a variety of other homological link invariants, including the Heegaard Floer homology of the branched double cover and instanton knot homology. However, there is no known relationship between Khovanov homology and knot Floer homology, despite considerable computational evidence and numerous formal similarities. I will describe ongoing efforts to find a spectral sequence relating these two invariants. Specifically, we construct a variant of Khovanov homology for links with one or more basepoints on each component, which more closely parallels the behavior of knot Floer homology and which conjecturally fits into a spectral sequence as required. This is joint work with John Baldwin and Sucharit Sarkar.

Gregor Masbaum: An application of TQFT to modular representation theory

Let $G = \mathrm{Sp}(2n, k)$ be a symplectic group defined over an algebraically closed field k of characteristic $p > 0$. Simple G -modules in the natural characteristic can be classified up to isomorphism by highest weights, but their dimensions are largely unknown in general. In this talk, we will present a new family of highest weights for G whose dimensions we can compute explicitly. The corresponding G -modules appear as a byproduct of Integral Topological Quantum Field Theory, and their dimensions are given by formulae similar to the Verlinde formula from Conformal Field Theory.

Catherine Meusburger: Mapping class group actions in Hopf algebra gauge theory

Group valued lattice gauge theories can be generalised to lattice gauge theories on a ribbon graph with values in finite-dimensional ribbon Hopf algebra. The resulting model is equivalent to the quantum moduli algebra from the combinatorial quantisation of Chern-Simons theory and, for Drinfeld doubles of semisimple Hopf algebras, also to Kitaev lattice models. We derive a simple description of the mapping class group actions in these models that describes Dehn twists as well as the mapping class group action on trivalent graphs via flip moves. The results do not need semisimplicity.

Cristina Palmer-Anghel: A homological model for the colored Jones polynomials

In 1991, Reshetikhin and Turaev defined a method that starts with a quantum group and leads to link invariants. This construction is purely algebraic and combinatorial. The coloured Jones polynomials $J_N(L, q)$ are a family of quantum invariants constructed from the representation theory of $U_q(\mathfrak{sl}(2))$. We will describe a geometrical interpretation for the coloured Jones polynomials.

R. Lawrence defined a sequence of representations of the braid groups, using the homology of a certain covering of a configuration space. In 2012, Kohno proved a deep connection between the representations of the braid group on the highest weight spaces of $U_q(\mathfrak{sl}(2))$ -modules and the Lawrence representations. Using this, we give a homological model for $J_N(L, q)$. We prove that the coloured Jones polynomials can be described as a graded intersection pairing between two homology classes on a covering of the configuration space of the punctured disc.

Ina Petkova: Knot Floer homology and the $\mathfrak{gl}(1|1)$ link invariant

The Reshetikhin-Turaev construction for the standard representation of the quantum group $\mathfrak{gl}(1|1)$ sends tangles to $\mathbb{C}(q)$ -linear maps in such a way that a knot is sent to its Alexander polynomial. After a brief review of this construction, I will give an introduction to tangle Floer homology – a combinatorial generalization of knot Floer homology which sends tangles to (homotopy equivalence classes of) bigraded dg bimodules. Finally, I will discuss how to see tangle Floer homology as a categorification of the Reshetikhin-Turaev invariant. This is joint work with Alexander Ellis and Vera Vertesi.

Krzysztof Putyra: Quantized homology of colored knots

There are at least three ways to define $\mathfrak{sl}(2)$ link homology for knots colored by a symmetric representation:

- (1) the original construction due to Khovanov,
- (2) using categorified Jones-Wenzl projectors due to Cooper and Krushkal,
- (3) as the image of a certain sum of maps induced by cobordisms, mimicking the formula for the Jones-Wenzl projector in the Temperley-Lieb category.

They produce different complexes when applied to the usual $\mathfrak{sl}(2)$ -homology. For instance, (1) is finite but (2) has infinite homology. On the other hand, all three coincide, when the quantized $\mathfrak{sl}(2)$ -homology, constructed by Anna Beliakova, Stephan Wehrli, and myself, is used. In particular, the quantum closure of the Jones-Wenzl projector is homotopy equivalent to a finite complex. In my talk I will describe how the quantum homology arises, explain why we called it quantum, and give some insight into the proof of equivalence of the three constructions mentioned before.

This is a joint project with Anna Beliakova, Stephan Wehrli, and Matt Hogancamp.

Alexis Virelizier: Generalized Kuperberg invariants of 3-manifolds

In the 90s, Kuperberg defined a scalar invariant of 3-manifolds from each finite-dimensional involutory Hopf algebra over a field. The construction is based on the presentation of 3-manifolds by Heegaard diagrams and involves tensor products of the structure tensors of the Hopf algebra. These tensor products are then contracted using integrals of the Hopf algebra to obtain the scalar invariant. We generalize this construction by contracting the tensor products with other morphisms. Examples of such morphisms are derived from involutory Hopf algebras in symmetric monoidal categories. This is a joint work with R. Kashaev.

Emmanuel Wagner: Foams and categorification

We will discuss various categorifications of quantum link invariants like Jones polynomials, $\mathfrak{sl}(n)$ polynomials and colored Jones polynomials using foam technology. All the constructions will be based on a combinatorial formula for closed foams and universal constructions. This is based on joint works with L-H. Robert.

Paul Wedrich: On categorification of skein modules and algebras

Khovanov homology and its cousins are usually defined as functorial invariants of links in R^3 . Embracing their reliance on link projections as a virtue, they admit an extension to links in thickened surfaces, and, thus, categorify surface skein modules and, conjecturally, their algebra structures. Skein algebras are related to character varieties and quantum Teichmüller theory, and are the subject of positivity conjectures that appear in reach of categorification techniques. The focus of this talk will be recent joint work with Hoel Queffelec on functorial $\mathfrak{gl}(2)$ surface link homologies.

Michael Willis: Colored Khovanov-Rozansky homology for infinite braids

We show that the limiting unicolored $\mathfrak{sl}(N)$ Khovanov-Rozansky chain complex of any infinite positive braid categorifies a highest-weight projector. This result extends an earlier result of Cautis categorifying highest-weight projectors using the limiting complex of infinite torus braids. Additionally, we show that the results hold in the case of colored HOMFLY-PT Khovanov-Rozansky homology as well.

Joint with Michael Abel.